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Stripwise Calculation of Hydrodynamic Forces Due to Beam Seas

By Seizo Motora[1]

A method has been developed for calculating the oscillatory heave and sway force on a surface ship at zero speed in beam regular waves, using an extension of the strip theory of Korvin-Kroukovsky and Jacobs. Calculations were made using the damping and inertia coefficients of Tasai for the Series-60, 0.60 block coefficient ship model and very good agreement was found with experimental measurements at Davidson Laboratory and with theoretical values of Newman. Comparison of calculated wave forces for a cylinder with exact solutions of Ursell and Newman has also shown reasonable agreement.

THIS note is an extension of Korvin-Kroukovsky and Jacobs' method [1][2] of evaluating the heaving force due to head seas into the evaluation of heaving and swaying force of a ship due to beam seas. A new attempt is made to take into account the effect of damping.

In this paper, heaving force and swaying force are each expressed as a sum of an inertia term, a damping term and a buoyancy term, and each term is corrected for the effect of the orbital motion of the wave. This method of expression was suggested by Jacobs [2] and it was also shown by Hu [3] that lateral force due to beam seas is directly related to the added mass of the body.

Since the virtual inertia coefficient and damping coefficient are calculated for various section forms as frequency-dependent functions by Ursell [4], Grim [5], and Tasai [6, 7] it will be convenient to use this method.

Fundamental Assumption

Suppose a wave train of height $2h$ travelling to the $+y$-direction impinges on a fixed body whose axis is parallel to the wave crest line, Fig. 1.

Within the linear range, the wave motion is the vector sum of up-and-down motion and back-and-forth motion with 90 deg phase difference. Therefore, if wave elevation is

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[2] Numbers in brackets designate References at the end of the paper.

Nomenclature

$\bar{A}$ = amplitude ratio, generated wave/body oscillation
A = waterplane area per unit length
a = cylinder radius
B^* = ship midship beam
C = coefficient
c = wave celerity
e = naperian
F = total hydrodynamic force per unit length
g = acceleration of gravity
h = incident wave amplitude
K = wave parameter = ω^2/g
k_y, k_z = virtual inertia coefficients in sway and heave, respectively
L = cylinder or ship length
N_y, N_z = sway and heave damping coefficients, respectively
P = pressure
r = radial body coordinate
S = surface area
t = time
T = ship draft
V = displacement volume per unit length
v = velocity
W = displacement of ships
y, z = space coordinates
y_w = instantaneous horizontal displacement of surface water particle
Z = instantaneous heave displacement
Z_w = instantaneous wave elevation
α = angular body coordinate
$\gamma_{1z}, \gamma_{2z}, \gamma_{3z}$ = correction factors for inertia, damping, and buoyancy forces in heave, respectively
$\gamma_{1y}, \gamma_{2y}, \gamma_{3y}$ = correction factors for sway forces
η = amplitude of body-generated wave
λ = wave length
ρ = density
ϕ = total velocity potential
$\phi_{bm}, \phi_{bw}, \phi_w$ = velocity potentials due to body motion, body wave interaction, and wave motion, respectively
ω = circular frequency

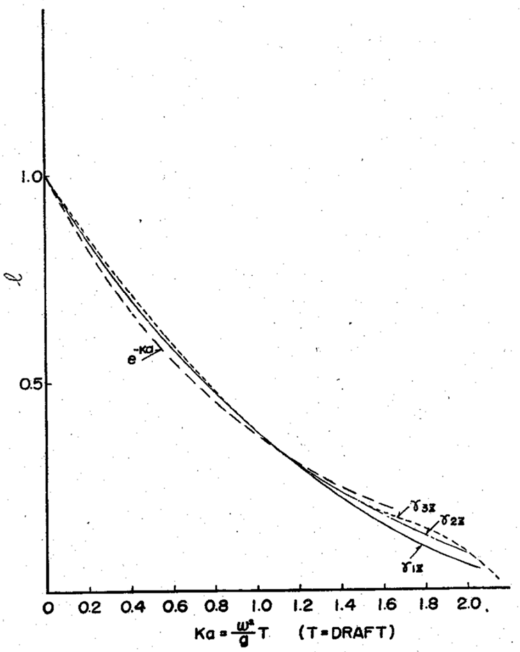

Fig. 1 Correction factors for heaving force

$$Z_w = h \sin(Ky - \omega t)$$

where

$K = 2\pi/\lambda = \omega^2/g$
ω = circular frequency of wave
h = wave amplitude

then, at the origin

Up-and-down motion, $Z_w = -h \sin\omega t$
Back-and-forth motion, $y_w = h \cos\omega t$ (1)

On the other hand, if the body heaves with amplitude Z and frequency ω, total hydrodynamic force will be expressed as follows:

$$F_z = \rho V k_z \frac{d^2 z}{dt^2} + N_z \frac{dz}{dt} + \rho g A Z \quad (2)$$

where

V = displacement volume per unit length
k_z = virtual inertia coefficient
N_z = damping coefficient
A = waterplane area per unit length

The first and the third terms are in phase with the motion, and the second term is out of phase.

Replacing Z by Z_w, and introducing correction factors γ_{1z}, γ_{2z}, and γ_{3z}, for the effect of orbital motion of the wave, we get

$$F_{zw} = \gamma_{1z} \rho V k_z \frac{d^2 Z_w}{dt^2} + \gamma_{2z} N_z \frac{dZ_w}{dt} + \gamma_{3z} \rho g A Z_w \quad (3)$$

γ_{3z} has been known as the Smith's correction factor. The first and third terms are in phase with the wave elevation, and the second term is out of phase with the wave elevation.

In a similar manner, swaying force can be described as

$$F_{yw} = \gamma_{1y} \rho V k_y \frac{d^2 y_w}{dt^2} + \gamma_{2y} N_y \frac{d_{yw}}{dt} + \gamma_{3y} \rho g V \frac{\partial Z_w}{\partial y} \quad (4)$$

The third term represents the horizontal component of the buoyancy. As y_w is in 90 deg advance of Z_w, F_{yw} is in 90 deg advance of F_{zw}.

Calculation of Correction Factors

Correction Factors for Heaving Force

Similar to Korvin-Kroukovsky's method [1] velocity potential around the body is expressed as

$$\phi = \phi_{bm} + \phi_{bw} + \phi_w \quad (5)$$

where ϕ_{bm} is potential due to the motion of the body which is now zero, ϕ_{bw} is potential due to body-wave interaction, and ϕ_w is potential due to wave motion alone.

Correction for Inertia Term. For simplicity, let us take a circular cylinder of radius a. Also assume that ϕ_{bw} is to be approximated using the potential function for a doublet in a uniform stream even though the body is actually in stream of decreasing velocity. Thus we obtain

$$\phi_{bw} = v_{wz} \frac{a^2}{r} \cos\alpha = \omega h \frac{a^2}{r} e^{KZ} \cos\alpha \cos(Ky - \omega t) \quad (6)$$

where

$$K = \frac{\omega^2}{g} = \frac{2\pi}{\lambda}$$

Inertia force is obtained by integrating pressure due to ϕ_{bw} around the surface of the cylinder; Therefore

$$\text{inertia } F_{zw} = \rho \int_{-\pi/2}^{\pi/2} \frac{\partial \phi_{bw}}{\partial t} a \cos\alpha d\alpha$$
$$r = a$$
$$= \rho \omega^2 h a^2 \int_{-\pi/2}^{\pi/2} e^{-Ka\cos\alpha} \sin(Ky - \omega t) \cos^2\alpha d\alpha \quad (7)$$

Owing to symmetry

$$F_{zw\ \text{inertia}} = -2\rho a^2 \omega^2 h \sin\omega t \int_0^{\pi/2} e^{-Ka\cos\alpha} \cos \times (Ka \sin\alpha) \cos^2\alpha d\alpha = -\frac{\rho\pi a^2}{2} \omega^2 h \sin\omega t \times \underbrace{\frac{4}{\pi} \int_0^{\pi/2} e^{-Ka\cos\alpha} \cos(Ka \sin\alpha) \cos^2\alpha d\alpha}_{\gamma_{1z}} \quad (8)$$

Since $\rho\pi a^2/2 = \rho V$, and $+\omega^2 h \sin\omega t$ is vertical acceleration of the wave surface $d^2 Z_w/dt^2$ equation (8) can be rewritten as

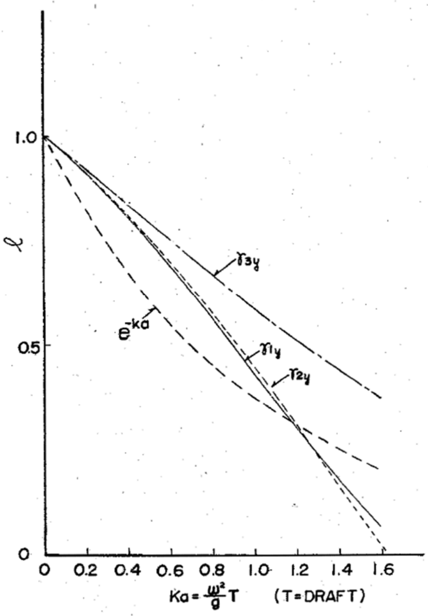

Fig. 2 Correction factors for swaying force

$$F_{zw\ \text{inertia}} = -\rho V \frac{d^2 Z_w}{dt^2} \gamma_{1z} \tag{9}$$

where

$$\gamma_{1z} = \frac{4}{\pi}\int_0^{\pi/2} e^{-Ka\cos\alpha}\cos(Ka\sin\alpha)\cos^2\alpha d\alpha$$

This term should be multiplied by $k_z' = k_2 k_4$ (Ursell's notation) to correct for the effect of free surface upon the added mass coefficient.

Therefore equation (9) becomes exactly the same form as equation (3), and γ_{1z} in equation (9) gives the correction factor

$$\gamma_{1z} = 1 - \frac{8}{3\pi}(Ka) - \tfrac{1}{4}(Ka) - \frac{4}{45\pi}(Ka)^3 + 0[(Ka)^5] \tag{10}$$

γ_{1z} is shown in Fig. 2 on Ka base.

Correction Factor for Buoyancy. The factor has been known as Smith's correction factor. Buoyancy will be obtained by integrating pressure caused by the wave itself over the body's surface.

$$F_{z\ \text{buoyancy}} = \rho\int_{-\pi/2}^{\pi/2} \frac{\partial\phi_w}{\partial t} a\cos\alpha d\alpha \tag{11}$$

Since

$$\phi_w = hce^{KZ}\cos(Ky - \omega t) \tag{12}$$

where

$$c = \text{wave celerity}$$

$$F_{z\ \text{buoyancy}} = \rho\int_{-\pi/2}^{\pi/2} hc\omega a e^{KZ}\sin(Ky - \omega t)\cos\alpha d\alpha$$
$$= \rho g a h\int_{\pi/2}^{\pi/2} e^{KZ}\sin(Ky - \omega t)\cos\alpha d\alpha \tag{13}$$

Because of symmetry

$$F_{zb} = -2\rho g a h\sin\omega t\int_0^{\pi/2} e^{KZ}\cos(Ka\sin\alpha)\cos\alpha d\alpha \tag{14}$$

Since $2a = A$, the waterplane area per unit length, $2\rho gah$ corresponds to the change of buoyancy between a wave trough and a wave crest. Therefore equation (14) can be written as

$$F_{zb} = -\rho g A h\sin\omega t\gamma_{3z} = \gamma_{3z}\rho g A Z_w$$
$$\gamma_{3z} = \int_0^{\pi/2} e^{-Ka\cos\alpha}\cos(Ka\sin\alpha)\cos\alpha d\alpha \tag{15}$$

This is an in phase force also, and γ_{3z} is expanded as follows:

$$\gamma_{3z} = 1 + \frac{\pi}{4}(Ka) + \tfrac{1}{6}(Ka)^2 + \frac{\pi}{360}(Ka)^4 + 0[(Ka)]^5 \tag{16}$$

γ_{3z} is shown in Fig. 1 on Ka basis.

Correction Factor for Damping. The damping-force coefficient N_z in equation (2) is defined as an out-of-phase force, which causes formation of progressive waves. These waves carry the energy away from an oscillating body, i.e., cause dissipation of energy. Designating the amplitude of the progressive wave by η, and the ratio of this amplitude to the body's oscillation amplitude z by $\bar{A}$, the damping-force coefficient N_z is evaluated as

$$N_z = \frac{\rho g \bar{A}^2}{\omega^3}$$

The ratio $\bar{A}$ was evaluated by Holstein [8], Havelock [9], Grim [5, 10], and Tasai [6], for a body making heaving oscillations in smooth water. In this case the undisturbed water flow relative to the body is the same at all depths. This evaluation can also be used in the case of a water surface rising and falling as a train of waves passes a restrained (stationary) body. In this latter case, however, the velocity of the orbital water motion is not constant but diminishes exponentially with depth. The force resulting from the wave amplitude η, is therefore smaller than that due to an equal body oscillation amplitude z. The coefficient N_z in equation (2) is therefore replaced by $\gamma_{2z}N_z$ in forming the wave force equation (3), where γ_{2z} is the correction factor. The following text will be devoted to the evaluation of this correction factor.

First, let us assume that the square of the amplitude of the scattered wave is proportional to the impulse given by the presence of the ship at the surface of the ship. And since the contribution of the impulse at a deep position should be less than that at a shallow position, let us assume that the contribution of each impulse to the scattered wave is proportional to an exponential of depth times K; viz.

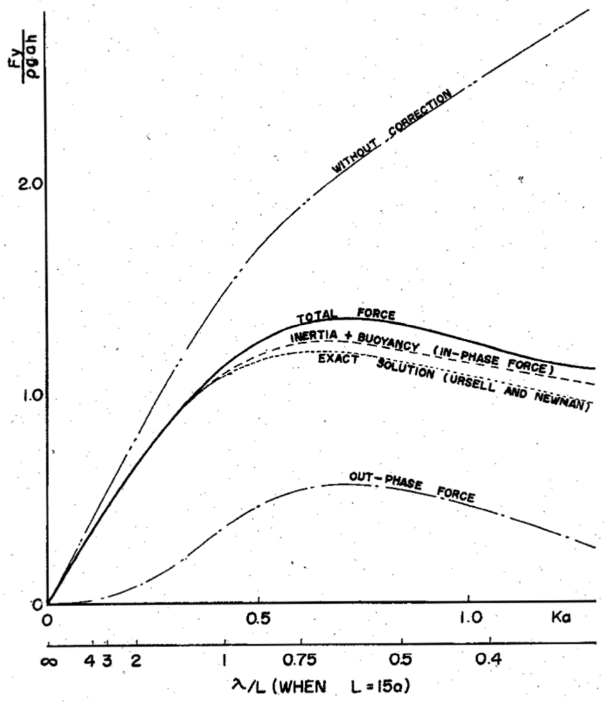

Fig. 3 Swaying force

$$\Delta\eta^2 \alpha \Delta S \int p e^{KZ} dt = \rho\phi e^{KZ} \Delta S \qquad (17a)$$

where ΔS is a differential element of area on the ship's surface.

Next, let us consider two points which are on the surface of water at the distance of $\pm y_0$ from the origin. Impulses acting at these points will create symmetric wave trains; one is propagating in the $+y$-direction and the other in the $-y$-direction.

Waves created at point A are

$$\eta_1 = \bar{\eta} \cos(\omega t \pm Ky + Ky_0)$$
$$- \text{ for } +y\text{-direction}$$
$$+ \text{ for } -y\text{-direction}$$

Waves created at point B are

$$\eta_2 = \bar{\eta} \cos(\omega t \pm Ky - Ky_0)$$
$$- \text{ for } +y\text{-direction}$$
$$+ \text{ for } -y\text{-direction}$$

In the case of heaving, η_1 and η_2 are in phase; Therefore

$$\eta = \eta_1 + \eta_2 = 2\bar{\eta} \cos(\omega t \pm Ky) \cos Ky_0$$

In the case of swaying η_1 and η_2 are 180 deg out of phase; Therefore

$$\eta = \eta_1 - \eta_2 = -2\eta \sin(\omega t \pm Ky) \sin Ky_0$$

The square of the amplitude of the scattered wave is

$$|\eta|^2 = 4\bar{\eta}^2 \cos^2 Ky_0 \quad \text{for heave}$$
$$|\eta|^2 = 4\bar{\eta}^2 \sin^2 Ky_0 \quad \text{for sway} \qquad (17b)$$

From (17*a*) and (17*b*) we get

$$\Delta|\eta|^2 = \rho\phi e^{KZ} \cos^2 Ky_0 \Delta S \qquad \text{for heave}$$
$$\Delta|\eta|^2 = \rho\phi e^{KZ} \sin^2 Ky_0 \Delta S \qquad \text{for sway}$$

Therefore

$$N_z \backsim |\eta^2| \alpha \int \rho\phi e^{KZ} \cos^2 Ky_0 dS \qquad \text{for heave}$$
$$N_y \backsim |\eta|^2 \alpha \int \rho\phi e^{KZ} \sin^2 Ky_0 dS \qquad \text{for sway} \qquad (18)$$

Introducing a certain unknown coefficient C, we get:

For generated wave by heaving of a ship

$$\eta_{bm}^2 = C \int \rho\phi_{bm} e^{KZ} \cos^2 Ky_0 dS \qquad (19)$$

For scattered wave by body-wave interaction

$$\eta_{bw}^2 = C \int \rho\phi_{bw} e^{KZ} \cos^2 Ky_0 dS$$

Therefore, correction factor γ_{2z} is given by

$$\gamma_{2z} = \frac{\eta_{bw}^2}{\eta_{bm}^2} = \frac{\int \phi_{bw} e^{KZ} \cos^2 Ky_0 dS}{\int \phi_{bm} e^{KZ} \cos^2 Ky_0 dS} \qquad (20)$$

Substituting

$$\phi_{bm} = v_z \frac{a^2}{r} \cos\alpha$$
$$= h\omega a \cos\omega t \cos\alpha \text{ at } r = a$$

and

$$\phi_{bw} = h\omega a e^{KZ} \cos(Ky - \omega t) \cos\alpha \quad \text{at } r = a \qquad (21)$$

into equation (20) we get

$$\gamma_{2z} = \frac{\int_0^{\pi/2} e^{-2Ka\cos\alpha} \cos^3(Ka \sin\alpha) \cos\alpha d\alpha}{\int_0^{\pi/2} e^{-Ka\cos\alpha} \cos^2(Ka \sin\alpha) \cos\alpha d\alpha} \qquad (22)$$

γ_{2z} is shown in Fig. 2.

Correction Factor for Swaying Force

Correction for Inertia Terms. The additional velocity potential due to body-wave interaction assumes a form

$$\phi_{bw} = v_{yw} \frac{a^2}{r} \sin\alpha$$
$$= \omega h \frac{a^2}{r} e^{KZ} \sin(Ky - \omega t) \sin\alpha \qquad (25)$$

Therefore

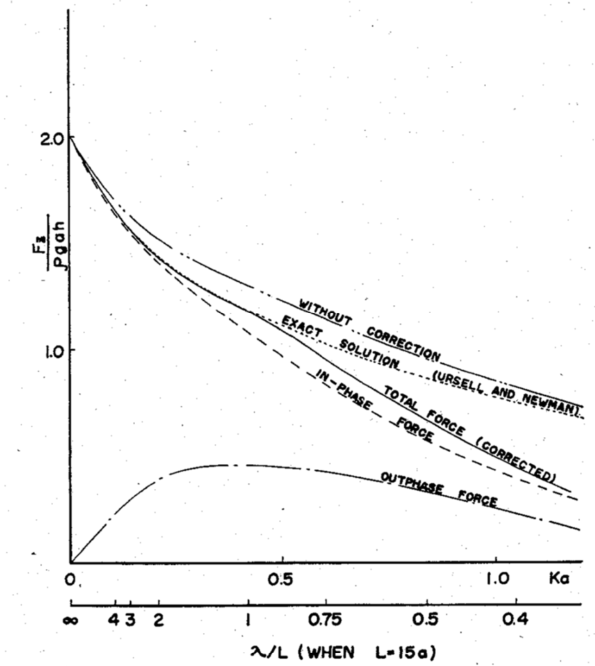

Fig. 4 Heaving force

$$F_{yw\ \mathrm{inertia}} = \rho \int_{-\pi/2}^{\pi/2} \frac{\partial \phi_{bw}}{\partial t}\, a \sin\alpha d\alpha$$

$$= -\rho a^2 \omega^2 h \int_{-\pi/2}^{\pi/2} e^{KZ} \cos(Ky - \omega t) \sin^2\alpha d\alpha \quad (26)$$

Because of symmetry

$$F_{yw\ \mathrm{inertia}} = -\frac{\pi \rho a^2 \omega^2 h}{2} \cos\omega t \times \frac{4}{\pi} \int_0^{\pi/2} e^{-Ka\cos\alpha} \cos(Ka \sin\alpha) \sin^2\alpha d\alpha \quad (27)$$

Since $\omega^2/g = 2\pi/\lambda$, and $(2\pi h/\lambda) \cos \omega t$ = wave slope, equation (27) can be rewritten as follows:

$$F_{yw\ \mathrm{inertia}} = \rho g V \times \text{wave slope} \times \gamma_{1y} \quad (28)$$

where

$$\gamma_{1y} = \frac{4}{\pi} \int_0^{\pi/2} e^{-Ka\cos\alpha} \cos(Ka \sin\alpha) \sin^2\alpha d\alpha \quad (29)$$

Equation (28) should be corrected for free surface effect by multiplying by k_y. Thus we get

$$F_{yw\ \mathrm{inertia}} = -k_y \rho g V \times \text{wave slope} \times \gamma_{1y} \quad (30)$$

As shown later, the third term becomes

$$F_{yw\ \mathrm{buoyancy}} = -\rho g V \times \text{wave slope} \times \gamma_{3y} \quad (31)$$

Thus, the inertia term almost doubles the swaying force. γ_{1y} is expanded as follows and is shown in Fig. 3:

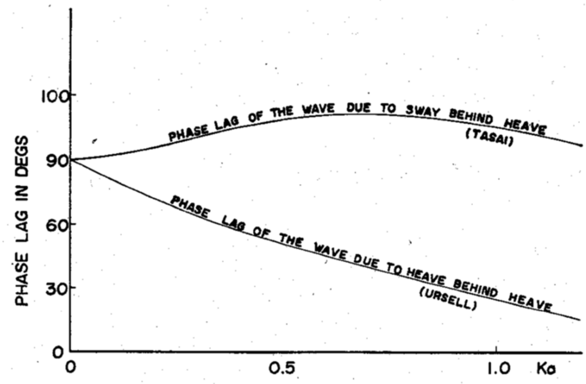

Fig. 5

$$\gamma_{1y} = 1 - \frac{4}{3\pi}(Ka) - \frac{(Ka)^2}{4} + \frac{14}{45\pi}(Ka)^3 + \ldots\ldots 0[(Ka)^5] \quad (32)$$

Correction for Buoyancy Term. The buoyancy term is obtained by integrating pressure over the ship's surface:

$$F_{yw\ \mathrm{buoyancy}} = \int p \sin\alpha dS = \rho \int \left(\frac{\partial \phi_w}{\partial t}\right) \sin\alpha dS \quad (33)$$

Substituting $\phi_w = hce^{KZ} \cos(Ky - \omega t)$

$$\frac{\partial \phi_w}{\partial t} = hc\omega e^{KZ} \sin(Ky - \omega t)$$

into equation (33) we get

$$F_{yw,b} = \rho h c \omega a \int_{-\pi/2}^{\pi/2} e^{KZ} \sin(Ky - \omega t) \sin\alpha d\alpha \quad (34)$$

Owing to symmetry, and since $c\omega = g$

$$F_{yw,b} = -2\rho g a h \cos\omega t \int_0^{\pi/2} e^{-Ka\cos\alpha} \times \sin(Ka \sin\alpha) \sin\alpha d\alpha \quad (35)$$

The integral is expanded as follows:

$$\int_0^{\pi/2} e^{-Ka\cos\alpha} \sin(Ka \sin\alpha) \sin\alpha d\alpha$$

$$= \frac{\pi}{4} Ka \left[1 - \frac{4}{3\pi}(Ka) + \frac{2}{45\pi}(Ka)^3 + 0\{(Ka)^4\}\right] \quad (36)$$

Therefore, equation (35) is written as

$$F_{yw,b} = -\frac{\pi \rho g a^2 K h}{2} \cos\omega t\ \gamma_{3y}$$

$$= -\frac{\pi \rho g a^2}{2} \frac{2\pi h}{\lambda} \cos\omega t\ \gamma_{3y} \quad (37)$$

$$= -\rho V \times \text{wave slope} \times \gamma_{3y}$$

where

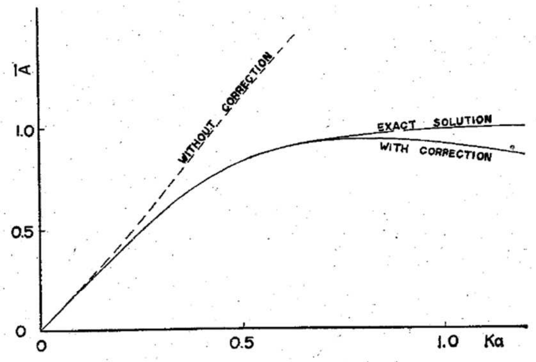

Fig. 6

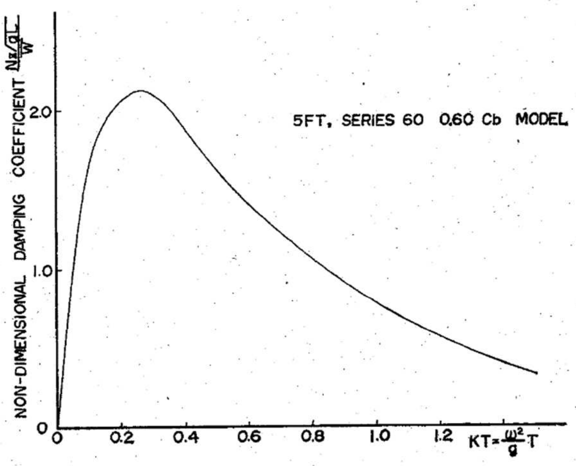

Fig. 8 Heaving damping coefficient

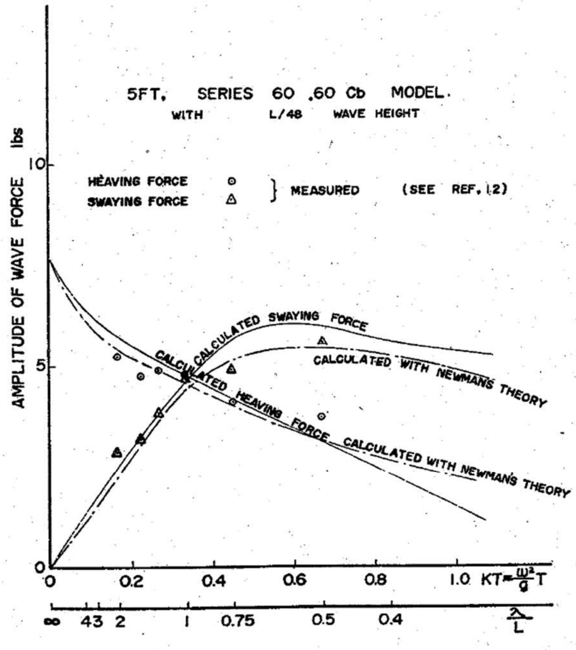

Fig. 7 Amplitude of oscillatory wave force

$$\gamma_{3y} = 1 - \frac{4}{3\pi}(Ka) + \frac{2}{45\pi}(Ka)^3 + 0\{(Ka)^5\} \quad (38)$$

This is the in-phase component, and as shown in "Correction for Inertia Term," this term has the same nature and the same sign as the inertia term given by equation (30).

γ_{3y} is shown in Fig. 3 on Ka basis.

Correction for Damping Term. Similar to "Correction Factor for Damping," let us assume that the square of amplitude of the scattered wave is proportional to the impulse given on (at) the surface of the ship. Therefore, from equations (17*b*) and (18) and similar to equation (20), we get

$$\gamma_{2y} = \frac{{\eta_{bw}}^2}{{\eta_{bm}}^2} = \frac{\int \phi_{bw} e^{KZ} \sin^2 Ky_0 dS}{\int \phi_{bm} e^{KZ} \sin^2 Ky_0 dS} \quad (39)$$

Substituting

$$\phi_{bm} = -v_y \frac{a^2}{r} \sin\alpha = -h\omega a \sin\omega t \sin\alpha \quad \text{at } r = a$$

and

$$\phi_{bw} = -v_{yw} \frac{a^2}{r} \sin\alpha = -n\omega a e^{KZ} \sin(Ky - \omega t) \sin\alpha \quad (40)$$

into equation (39) we get

$$\gamma_{2y} = \frac{\int_0^{\pi/2} e^{-2Ka\cos\alpha} \cos(Ka\sin\alpha) \sin^2(Ka\sin\alpha) \sin\alpha d\alpha}{\int_0^{\pi/2} e^{-Ka\cos\alpha} \sin^2(Ka\sin\alpha) \sin\alpha d\alpha} \quad (41)$$

γ_{2y} is shown in Fig. 3.

Comparison With Exact Solution

To evaluate the accuracy of this method, heaving force and swaying force of a circular cylinder are calculated by equations (3) and (4) and are compared with the exact value given by Ursell [11] and recently by Newman [13].

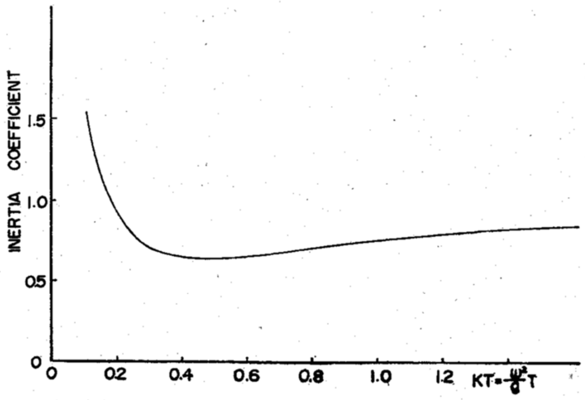

Fig. 9 Heave inertia coefficient

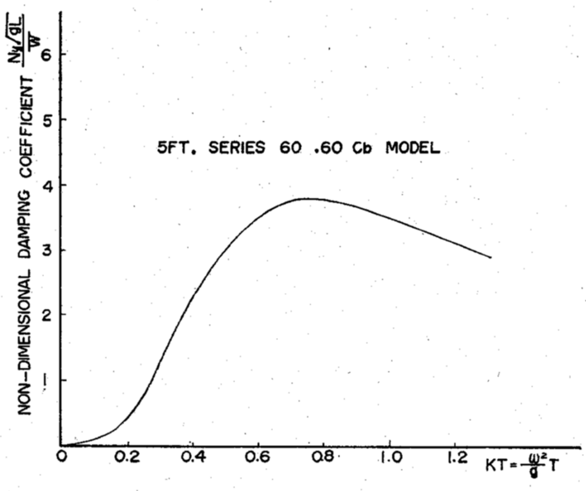

Fig. 10 Sway damping coefficient

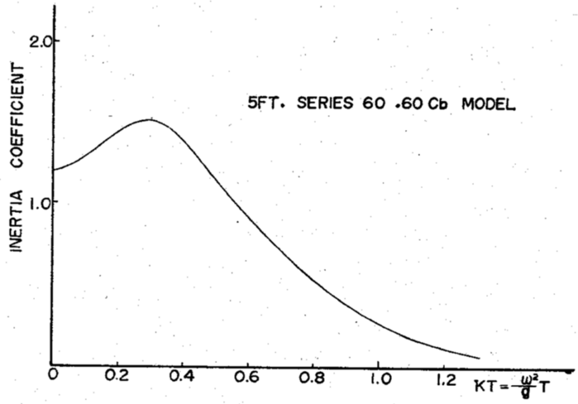

Fig. 11 Sway inertia coefficient

Ursell has calculated theoretically the amplitude of total heaving and total swaying forces acting on a semi-submerged circular cylinder fixed on the water surface. Ursell's results are shown in Figs. 4 and 5 by thin solid lines.

Recently Newman has shown that there are very simple relations between wave force and the amplitude of progressing wave caused by the motion of a body in still water. According to his expression, the amplitudes of heaving force and swaying force are given as follows:

$$F_z = \frac{\rho g h \bar{A}_z}{K} \tag{42}$$

$$F_y = \frac{\rho g h \bar{A}_y}{K} \tag{43}$$

where

$\bar{A}z$ = ratio of the amplitude of generated wave and amplitude of heaving

$\bar{A}y$ = ratio of amplitude of generated wave and amplitude of swaying.

Fz for a circular cylinder is thus calculated by equation (42) making use of $\bar{A}z$ calculated by Ursell [4] and Fy is also calculated by equation (43) making use of $\bar{A}y$ obtained by Tasai [7].

As seen in Figs. 4 and 5, results obtained by Newman's theory coincide with Ursell's results.

Fig. 4 shows the in-phase swaying force which is a sum of inertia (first term) and buoyancy (third term) of equation (4), and the out-of-phase force which is the second term of equation (4). A thick solid line shows the total force.

The total force estimated by this method agrees very well with the exact value when Ka is less than 0.4. When Ka is greater than 0.4 the estimated value still is in fairly good agreement with the exact value up to Ka = 1.3. If the cylinder is assumed to have typical ship proportions, i.e., length/beam = 7.5 or length = 15 × cylinder radius, then Ka = 0.4 corresponds to a wave length to ship length ratio, $\lambda/L = 1.0$. Thus, the foregoing comparisons show that this method is expected to agree very well for wave lengths longer than $\lambda/L = 1$ and fairly well for waves between $\lambda/L = 1.0$ and 0.3. By way of contrast, the dash-double-dot line in Fig. 4 shows the uncorrected force, i.e., when γ_{1y}, γ_{2y}, and γ_{3y} in equation (4) are all unity.

Fig. 5 shows the in-phase and out-of-phase heaving forces; the solid line represents the total force. The estimated value by this method agrees quite well with the exact value when Ka is less than 0.4 or when the wave length is longer than 15 times the cylinder radius. Agreement is still fairly good for wave lengths between 15 and 7.5 times the cylinder radius, or if cylinder length = 15 times radius, between $\lambda/L = 1.0$ and 0.5.

Fig. 5 also shows the heaving force without correction. Since the virtual inertia force and the buoyancy have a 180-deg phase difference, the effect of the correction is not as marked as in the case of swaying force. But for a section form other than a circular section—for instance, shallowdraft ship which will have larger virtual inertia and less buoyancy—the correction for the orbital motion should be more effective.

Amplitude of Reflected Wave

Amplitude of the reflected wave due to the presence of a circular cylinder was calculated theoretically by Ursell [11]. To examine whether the method of correcting the damping factor is reasonable, let us calculate the amplitude of the reflected wave by the present method and compare it with the exact solution.

When a body heaves with amplitude z, then the amplitude of the generated wave is given by

$$\eta_z = \bar{A}z \tag{44}$$

As described in "Correction Factor for Damping," it is assumed that, owing to the effect of orbital motion, the square of the amplitude of the wave generated by heaving of a body is reduced by a factor of γ_{2z} to obtain the square of the reflected wave amplitude.

$$\eta_{zw} = \gamma_{2z}^{1/2}\eta_z = \gamma_{2z}^{1/2}\bar{A}_z z \tag{45}$$

Similarly, we get for sway

$$\eta_{yw} = \gamma_{2y}^{1/2}\eta_y = \gamma_{2y}^{1/2}\bar{A}_y y \tag{46}$$

where

$\bar{A}_z$ = ratio of amplitude of generated wave and amplitude of heaving
$\bar{A}_y$ = ratio of amplitude of generated wave and amplitude of sway

η_{zw} and η_{yw} have a phase difference, as shown in Fig. 5.

Taking into account the phase difference and superposing them, we get approximate value of the amplitude of reflected wave, which is shown in Fig. 6 by a thick solid line.

Fig. 6 also shows the exact value calculated by Ursell. The approximate value agrees with the exact value very well when λ/L is larger than 0.5.

Wave Force of a Series 60 Ship Form

The heaving and swaying forces of a Series 60, 0.60 block, 5-ft model were calculated using the strip method and are shown in Fig. 7 together with experimental results obtained at Davidson Laboratory by Lalangas [12]. Results obtained with Newman's theory [13] are also shown in Fig. 7 for the comparison.

For the inertia and damping coefficients in heave, use is made of the calculated results from the strip method of Tasai [6]. These are given in Figs. 8 and 9. Those for swaying are calculated from [7] and are given in Fig. 10.

As seen in Fig. 7 agreement between calculated value by this method and measured values is fairly good and a reasonable agreement between calculated values and Newman's theoretical values is also recognized.

It is supposed from the trend shown in Figs. 3 and 4 that calculated values are slightly overestimated for swaying and underestimated for heaving in the short wave range.

Conclusions

1 It is shown that the wave force acting on cylindrical bodies resulting from beam seas is expressed by the summation of an inertia term, a damping term, and a buoyancy term. Correction factors for orbital motion of waves for each term are obtained.

2 Approximate value of wave force thus calculated was compared with Ursell's and Neuman's exact solution for a circular cylinder, and showed reasonable agreement for wave lengths greater than 7.5 times the radius of cylinder. Therefore, if ship length is 7.5 times the beam, this approximation should be reasonable when wave length is greater than half the ship length.

3 Wave force acting on a Series 60, 0.60 block model was calculated and compared with experimental data obtained at Davidson Laboratory.

The calculated results from the theory presented in this paper as well as from Newman's theory agree quite well with experimental data.

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